METHODS OF USING
ELECTRICAL ANALOGS
OF
PHYSICAL STRUCTURES

A DISCUSSION AND EXPLANATION

by

ELMER V. CARLSON
Knowles Electronics, Inc.

PREFACE

Some procrustean and possibly unfashionable positions are taken and adhered to without apology. If the reader feels differently, he is free to follow his bent. He will deserve his successes but receive no sympathy for the undertaking.

Some of the approaches that are discussed, if not novel, are at least rare. It is not maintained that this is "the way", only that it is "a way" and one that has been useful and successful, much of it over a number of years.

In applying this material, it should be kept in mind that one is using an analog. That the analog is built on the similarity of some of the equations used to describe the behavior of two (analogous) systems and that these equations do not necessarily describe all the pertinent behavior. There may be conditions in one of the systems that are not accounted for by the relationships to the other. This is a condition that should prompt the user to remain alert, as it may produce unexpected results.

Common sense as opposed to blind faith is necessary to success. If the results seem too good or too bad to be true, they probably are.

OBJECTIVE

The purpose is to set forth and explain a system of methods and procedures devised to assist the development and design process of electroacoustic transducers.

In the following:

1. The approach is intended to be friendly to engineering and design applications. This is particularly true when it comes to flexibility in the choice of units.

2 The approach assumes a familiarity with electrical networks and the solution of the behavior of such networks as a common tool.

3. The "classical analogy" will be used:

   That is, sets of potential energy related parameters (voltage, pressure, force and magnetomotive force - which will be called "forcing" parameters) will be considered analogous as will the associated set of potential energy storing elements (capacitors, volumes, compliances and permeances).

   Kinetic energy related parameters (current, volume velocity, velocity and fluxion - which will be called "action" parameters) will be considered analogous as will the associated set of kinetic energy storage elements (inductances, inertances and masses).

   Those dissipative elements which unilaterally convert other energies to heat such as electrical resistance, viscosity and viscous friction are also analogous.

4. All non-linear and discontinuous behavior is excluded. Only systems that approach linear behavior, at least for small actions, are considered.

ANALOGS

By creating an electrical analog we mean that we fashion an electrical circuit where the condition of the circuit elements is imagined to represent behavior of elements in some other physical system. The voltages and currents will represent the forcings and actions of the corresponding physical elements. The systems of units in the analog are almost arbitrary. Various systems of units will be used. The one chosen would usually be the one in which conventional measurements and calculations are made. By proper scaling the impedance level of portions of the electrical circuit it is possible to have the numeric value assigned to the electrical element be equal to the numeric value used to describe the property of that analogous element. Requiring this condition makes it easy to think of each part of the system in familiar terms. That is, if a 0.5 henry inductor is analogous to a 0.5 gram mass it becomes easy to associate the effect of changes in the electrical circuit with similar changes in the mechanical structure.

PREMISE

We will rely on the first law of thermodynamics to build our models. If energy enters a port of a system, it remains someplace in the system unless it exits another port. If it is converted to heat which is allowed to dissipate from the system such loss elements must be considered as resistive and are ports of the system. The fact that all the energy must be accounted for is the kernel for the construction of the analog.

If we place an interface between two parts of the analog forming two systems, the energy that passes into one side of the interface from one system in the analog, must simultaneously leave the other side of that interface into the second system.

UNITS

The restraints on the choice of units of measure to be simulated in various portions of the analog are as follows:

1. The unit of time must be common. We will use the second.
2. The unit of "forcing" multiplied by the unit of "action" must yield the unit of power (time rate of energy).
   - 2.1 Gravity units of "forcing" cannot be used, i.e., use dynes not grams, poundals not pounds.
3. The unit of "forcing" divided by the unit of "action" must yield the unit of impedance.

NOTATION

While in the long term a uniform system of notation is a goal, it may not be consistently implemented in all of this text. Various parts were written at substantially different times and use notations that seemed reasonable at the time and may not have been updated. The trend toward uniformity is currently being directed by the following concepts.

We will use a notation implying electrical parameters such as E, I, Q, L, C and R. The system that they refer to will be suggested by a first alpha subscript.

Electrical	usually omitted, occasionally 'e'
Mechanical	m (lower case)
Magnetic	M (upper case)
Acoustic	A (upper case)

The type of function may be distinguished further by a second alpha subscript.

At signal frequency	a (lower case)
At steady state (or quasi-steady state)	d (lower case)

A numeric subscript may follow to distinguish different entities in the circuit.

Thus a mechanical displacement in a first circuit that is not a function of the signal (i.e., a bias or offset for instance) would be:

$$Q_{md1}$$

and an acoustic volume velocity at signal frequency in a second circuit would be:

$$I_{Aa2}$$

These subscriptings may be ignored when there appears to be no need for these distinctions.

MAGNETISM

It is necessary to give special attention to how we are to represent a magnetic circuit in these analogs. The frequently used representation of magnetomotive force (MMF) as a voltage, a reluctance as a resistance and flux as a current is of no use. To do this would imply that in any permanently magnetized structure, there is a constant expending of energy and a heating of the resistor (reluctance), which is not the case and violates the premise on which this system is built.

The following concept is rare. I have found only a few allusions to the approach in print. Take as usual, MMF and electrical voltage as analogous. Reluctance is to be analogous to capacitive reactance (and permeability to dielectric constant.) This, which at first may not seem intuitive, follows as it represents an element for storing potential energy. Flux becomes the analog of electrical charge. Now the equations for electric and magnetic energy take a similar form.

$$W = E^2 C/2 \qquad W_M = E_M{}^2 C_M/8\ \mathrm{Pi}$$

or what is the same thing

$W = Q^2/(2C)$ $W_M = Q_M^2/(8 \text{ Pi } C_M)$

This last form would usually appear in a more conventional notation as

Energy = $Ø^2/8$ Pi P where Ø = flux
P = permeance (i.e., A/l)

There seem to be no magnetic elements analogous to imperfect but linear conductors (i.e., resistors) or to inductors. Permeances (capacitive reactances), MMF's (voltages) and flux (charge) are the only circuit elements or parameters we have in magnetic circuits.

There is a need for a word to mean the rate-of-change of flux, to be analogous to an electrical current. For this purpose the use of the archaic word "fluxion" is revived, to be defined as the time derivative of magnetic flux.

The second unusual relationship we will adopt has to do with the magnetic units used. In this material an abridged form of the cgs system will be used. Where:

1. The unit of forcing (MMF), the Gilbert, will be analogous to the Volt.

2. The unit of displacement, the Line, will be analogous to the Coulomb.

3. The unit of action (fluxion), unnamed, will be analogous to the Ampere.

4. The unit of resistance, unnamed, will be analogous to the Ohm. This ratio, "unit forcing/unit action", or Gilberts/(Lines/Second), while useful analytically, does not appear to have any physical manifestation.

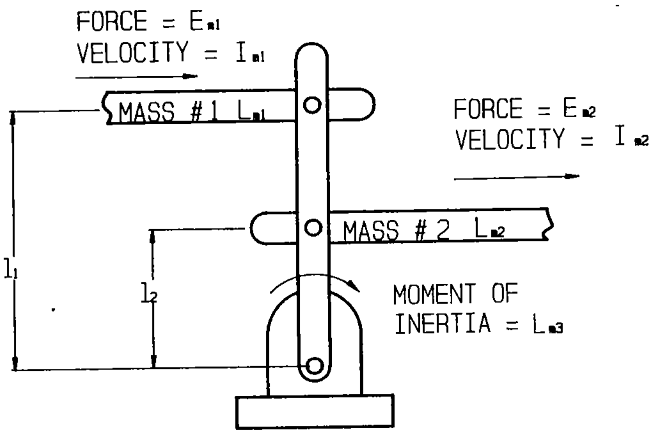
5. The unit for rate-of-energy, unnamed, will be analogous to the Watt. This product is "Gilberts x Lines-per-second" and is equal to 1/(4 Pi) ergs-per-second. The use of this unit removes the 4 Pi term from all equations except those involving energy, thus it will show up in these analogs only at points where the systems of units are being changed.

INTERFACES

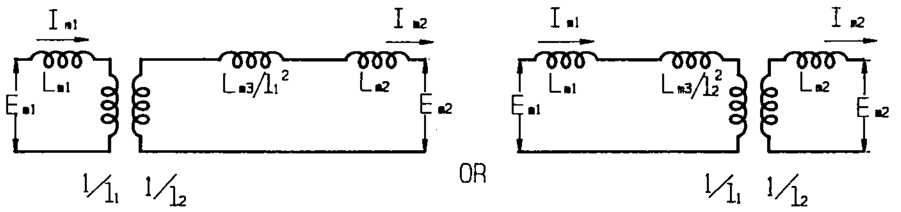
Between any two portions of the analog where the "forcings" or the "actions" are disjoint, there is a need for an interface. Mechanisms such as the lever, gear and crank are members of the general class of interfaces. In the above illustrations the energy is mechanical on both sides of the interface.
To simplify the definition of an interfacing element we consider, all parasitic elements are removed and relegated to the appropriate coupled systems. As an example:

The Lever as an Interface

In the case of a simple lever coupling linear motions:



The interface becomes an ideal transformer determining the force and velocity ratios established by the lever arm lengths. The moment of inertia of the lever structure can be referred to either side of the transformer or apportioned between the sides as best fits the problem.

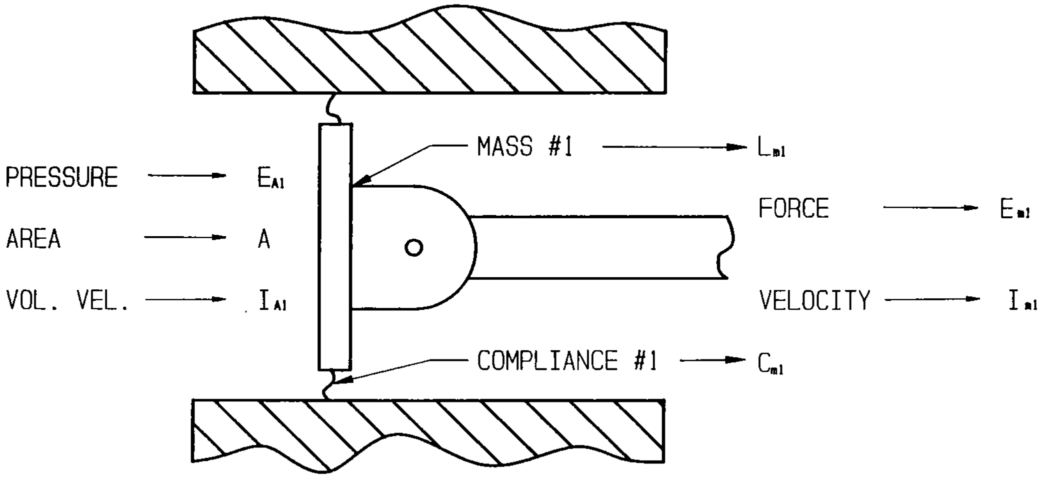


INTERFACES AS TRANSDUCING ELEMENTS

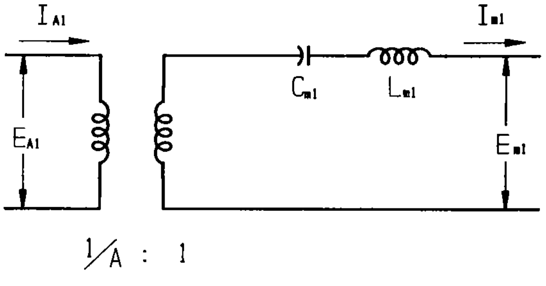
The "transducing element" is the name we apply to that interface where the energy passes from one portion of an analog system into another as a different form of energy. That is, by a transducer, we mean that the interface converts one form of energy into another. A diaphragm or bellows might be considered as being a transducer:

The Diaphragm (or Piston) as an Interface

The acoustic (or pneumatic) diaphragm is also modeled as a transformer where the effective area defines the ratio between the air pressure and the mechanical force:



In this case the mass of the structure and the stiffness of the supporting constraints are obvious mechanical parameters and would be assigned to the analog on the mechanical side of the interface.



There are devices frequently referred to as tranducers, that are not to be considered transducers in the sense we intend. These are active devices where an input of energy controls the rate at which energy is available from a second port. That energy being derived from a source attached at a third port. Examples of such devices are amplifiers, differential transformers and potentiometers.

The Electromagnet as a Transducer

One of the simplest interfaces that is a transducer is not usually shown explicitly in analog developments. This is the transformation between electrical and magnetic energies. The manner in which the magnetic parameters and elements have been defined permit us to make this construction. There are two equations that need to be satisfied:

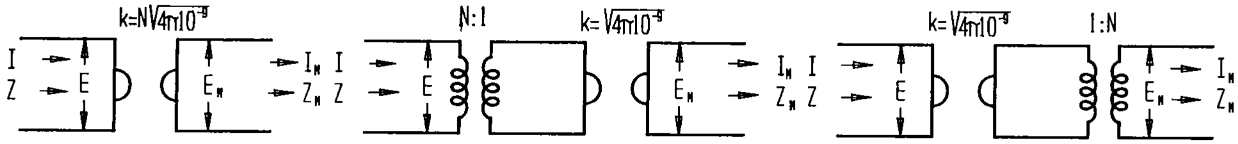
$$e = d(\emptyset N x 10^{-8})/dt \quad \text{--->} \quad E = I_M N x 10^{-8}$$

$$MMF = 4 \text{ Pi } \frac{NI}{10} \quad \text{--->} \quad E_M = .4 \text{ Pi } NI$$

By dividing:

$$\frac{E}{I} = \frac{4 \text{ Pi } N^2}{E_M} \frac{I_M}{10} \times 10^{-8} \quad \text{--->} \quad Z = \frac{4 \text{ Pi } N^2 \times 10^{-9}}{Z_M}$$

The systems are related by a gyrator with the factor $k = N(4 \text{ Pi } 10^{-9})^{.5}$ or .0001121 N. For convenience the factor N can be separated as a transformer on either side of the gyrator. (See Appendix B)



The gyrator factor does not have the property of "sided-ness" that is characteristic of the transformer ratio. For a review of the relative meaning of transformer ratios and gyrator factors see Appendix A.

This is a good point to note the effect of units on interfaces and the interpretation of the results. For the lever and the diaphragm the units of energy on both sides the interface would frequently (but not necessarily) be the same. That is, for the lever, voltage could be correlated with dynes force on both sides of the device and the behavior determined directly from values measured in the electrical analog. With the diaphragm, the same would apply for dynes and dynes/cm^2. The energy would be in ergs on both sides. In the case of the electromagnet the problem is more complicated. On the electrical side, the unit of energy is likely to be the Joule, while in the magnetic circuit, by our convention, the unit of energy is 1/(4 Pi) of an erg. The voltage in the magnetic portion of the analog will not directly read as magnetomotive force. An additive constant of 79 dB is necessary to make the voltage correspond to Gilberts. See the section on Calibrations.

In the more usual case, a transducer consists of a place where stored energy (usually potential energy) can be modulated by processes in either system and by this common factor produces the transduction energy from one system to the other. A more complicated transducer that is of this energy modulating type is the tractive air gap.

The Tractive Air Gap

Consider the similarity of the equations as set forth on pages 4 & 5 for energy in an electrical capacitor and in an magnetic air gap. These can be written as a single equation if we make some adjustments in the notation to insert the factors that don't appear explicitly and add the mechanical dimensions.

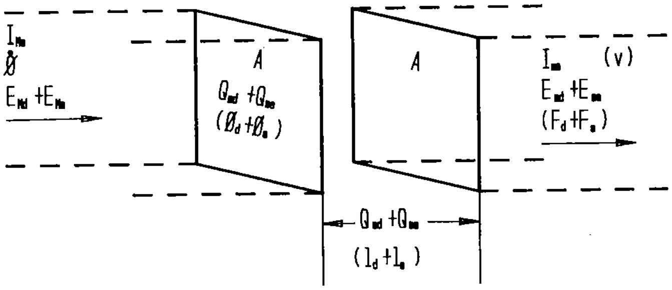
$$W = \frac{E_x^2 \; K_x \; A}{2Q_m} D_x \text{ ergs}$$

K_x is permeability, ($K_M = 1$), or permittivity, ($K_e = 8.85 \times 10^{-14}$), for air and relates the area (A) and the spacing (Q_m) to permeance or capacitance.

E_x is Gilberts (E_M) or Volts (E or E_e)

D_x is the dimensioning factor, $D_M = 1/(4 \text{ Pi})$ converting the magnetic unit of energy to ergs, or $D_e = 10^7$ converting the electrical unit of energy (the Joule) to ergs.

The magnetic (or electrical) properties of the construction that bring flux (or charge) to the air gap are parts of the magnetic (or electrical) circuits. The structural properties are a part of the mechanical circuit. Thus the tractive air gap consists solely of two juxtaposed surfaces to which are applied the magnetomotive (or electric) forces, magnetic flux (or electrical charge), mechanical motion and force. In this portion of the structure the energy stored in the included space can be changed by altering the strength of the magnetomotive force (or electrical voltage) or by mechanically changing the dimensions of the air gap.



Neglecting fringing, the energy stored is such an air gap is given by the relation:

$$W = \frac{(E_{xd} + E_{xa})^2 K_x A}{2(Q_{md} + Q_{ma})} D_x \text{ ergs}$$

The 'system' subscript of E_x is to be consistent with K_x and D_x.

Mechanical energy is force-over-a-distance, thus taking the derivative of the energy with respect to the separation of the surfaces yields the force between them.

$$E_m = \frac{dW}{dQ_{ma}} = \frac{d \; \frac{(E_{xd}^2 + 2E_{xd}E_{xa} + E_{xa}^2) \; K_x \; A}{(Q_{md} + Q_{ma})^2} \; D_x}{dQ_{ma}}$$

$$E_m = \frac{-(E_{xd}^2 + 2E_{xd}E_{xa} + E_{xa}^2) \; K_x \; D_x \; A}{2 \; Q_{md}^2 \; (1 + Q_{ma}/Q_{md})^2}$$

Expanding the denominator as a series and retaining only the linear terms we find the force components are:

$$E_m = - \frac{E_{xd}^2 \; K_x \; D_x \; A}{2 \; Q_{md}^2} - \frac{E_{xd} \; E_{xa} \; K_x \; D_x \; A}{Q_{md}^2} + \frac{E_{xd}^2 \; K_x \; D_x \; A \; Q_{ma}}{Q_{md}^3} \quad \text{dynes}$$

In this relation the first term is a static mechanical force and represents a steady pull that must be resisted by the mechanical system or the structure will collapse

The second term is a mechanical force that is related to the applied forcing at the signal frequency and will define the transduced signal.

The third term is linearly dependent on the spacing of the surfaces at signal frequencies, it thus represents a mechanical stiffness. The sign of this term is such that motion increases the amount of force in the direction of the motion. It is therefore a negative stiffness.

A second relationship exists in the air-gap relating the 'Forcing' and the mechanical spacing to the 'action' of an applied signal. At any instant we have:

$$Q_x = \frac{(E_{xd} + E_{xa}) \; A \; K_x}{Q_{md} + Q_{ma}}$$

Write I_x for $\frac{d\,Q_x}{d\,t}$

Take the derivative with respect to time to produce:

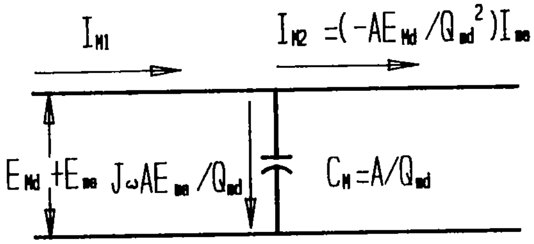
$$\frac{d\,Q_x}{d\,t} = I_x = \frac{-(E_{xd} + E_{xa})\,A\,K_x}{(Q_{md} + Q_{ma})^2}\,\frac{d\,Q_{ma}}{d\,t} + \frac{A\,K_x}{(Q_{ma} + Q_{md})}\,\frac{dE_{xa}}{d\,t}$$

as $\frac{d\,Q_{ma}}{d\,t} = I_{ma}$ and $\frac{d\,E_{xa}}{d\,t} = j\omega E_{xa}$

$$I_{xa} = -\frac{E_{xd}\,A\,K_x}{Q_{md}^2}\,I_{ma} + \frac{A\,K_x}{Q_{md}}\,j\omega E_{xa}$$

The first term provides a relationship between the applied action and the resultant mechanical velocity of the surfaces.

The second term represents a portion of the action that depends on the applied forcing and the dimensions. It becomes a circuit element associated with the transducer that now can be explicitly extracted. We recognize it as a shunt capacitance which, when introduced into the analog provides a simplified relationship of the action to the rest of the transducing element.



Rewrite the equation for the signal force from the middle of page 11 and the remaining portion of the above equation for velocity as follows:

$$E_{ma} - \frac{E_{xd}^2 \; A \; K_x \; D_x}{Q_{md}^3} Q_{ma} = - \frac{E_{xd} \; A \; K_x \; D_x}{Q_{md}^2} E_{xa}$$

$$I_{ma} = - \frac{Q_{md}^2}{E_{xd} \; A \; K_x} I_{xa}$$

divide these equations

$$\frac{E_{ma}}{I_{ma}} - \frac{E_{xd}^2 \; A \; K_x \; D_x}{Q_{md}^3} \frac{Q_{ma}}{I_{ma}} = \frac{E_{xd}^2 \; A^2 \; K_x^2 \; D_x}{Q_{md}^4} \frac{E_{xa}}{I_{xa}}$$

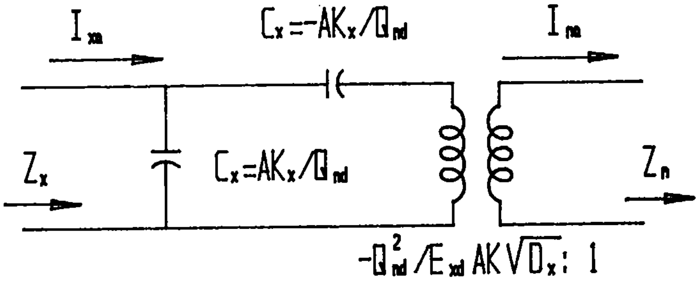
and rearrange

$$\frac{E_{xa}}{I_{xa}} = - \frac{Q_{md}}{A \; K_x} \frac{Q_{ma}}{I_{ma}} + \frac{Q_{md}^4}{E_{xd}^2 \; A^2 \; K_{x2} \; D_x} \frac{E_{ma}}{I_{ma}}$$

use the relation $I_{ma} = j\omega Q_{ma}$ and we have

$$Z_{xa} = - \frac{Q_{md}}{j\omega A \; K_x} + \frac{(Q_{md}^2)^2}{(E_{xd} \; A \; K_x)^2 \; D_x} Z_{ma}$$

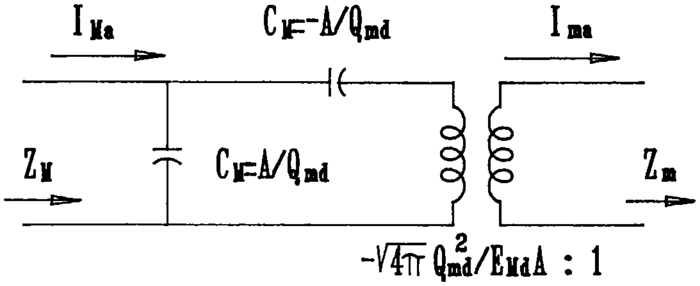
The first term on the right side represents a negative compliance in series with the transduced mechanical impedance. After we remove this as a explicit element, we can write the rest of the transducer as a transformer.



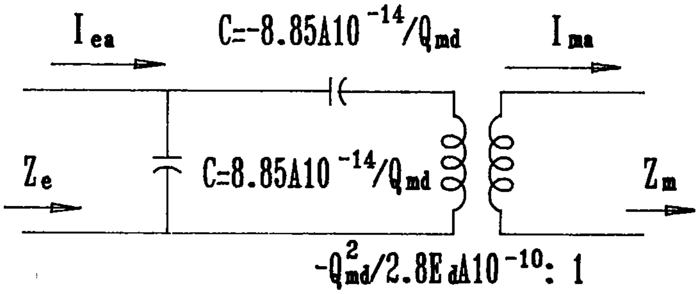
To conform with the equations at the middle of page 10 and the top of this page, choose the negative sign for the transformer ratio. That is when the signal forcing is in the same direction as the biasing forcing, the mechanical forcing works to produce a negative action. That is, to close the air gap.

This one relationship covers both the magnetic and the electrostatic tractive air-gaps. We have merely to replace x, Kx and Dx with the appropriate values.

In explicit form for the magnetic case:



and for the electrostatic case:



The static forces can be similarly calculated from the equation on page 11.

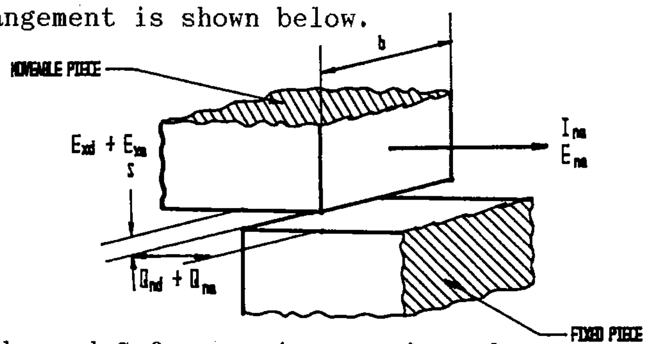
The two compliances depend only on the dimensions of the air gap and the coupling depends on these dimensions and the strength of the biasing field. There are several equivalent ways of writing the value of the transformer ratio. The following are frequently the simplest as they involve the generally known parameters.

$$-\frac{Q_{md}}{E_{Md}\ C_M}\,(4\ \mathrm{Pi})^{.5} \qquad\qquad -\frac{Q_{md}}{E_{ed}\ C_e}\,(10^{-7})^{.5}$$

The only place that the strength of the biasing appears is as a factor in this transformer ratio.

THE SHEAR AIR GAP

Another form of the energy modulation transducer is the air gap biased with an electric or magnetic field, where the area is modulated rather than the spacing. The schematic arrangement is shown below.



Two dimensions, b for breadth, and S for spacing are introduced so that, as on page 10, we can again write the equation for energy.

$$W = \frac{(E_{xd} + E_{xa})^2 \, (Q_{md} + Q_{ma}) \, b \, K_x \, D_x}{2 \, S} \text{ ergs}$$

When differentiated with respect to Q_{ma} and discarding the non-linear (distortion) terms we have:

$$E_m = \frac{b \, E_{xd}^2 \, K_x \, D_x}{2 \, S} + \frac{b \, E_{xd} \, K_x \, D_x}{S} \, E_{xa} \text{ dynes}$$

The first term is the steady force. The second term is the signal force. In this case neither term depends on the quiescent position (Q_{md}).

Rewriting the last relationship, on page 11, in terms of the present dimensioning we have:

$$Q_x = \frac{(E_{xd} + E_{xa}) \, (Q_{md} + Q_{ma}) \, b \, K_x}{S}$$

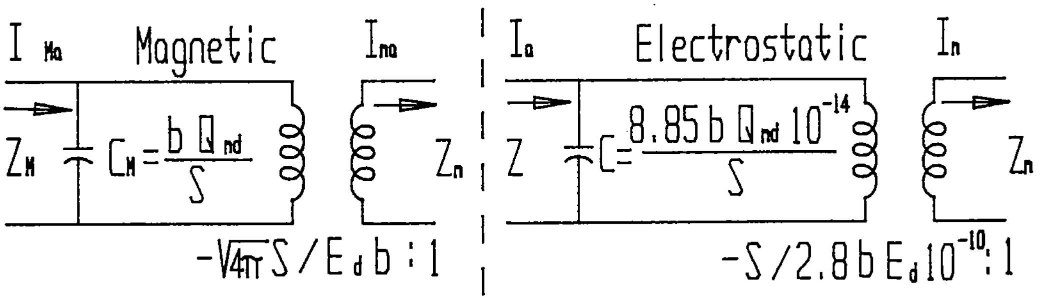
Differentiating with respect to time and discarding the distortion terms we obtain:

$$I_x = \frac{E_{xd}\ b\ K_x}{S}\ I_{ma} + \frac{J\quad Q_{md}\ b\ K_x}{S}\ E_{xa}$$

Again the second term represents a shunt capacitor which we remove as an explicit element. By dividing the remaining portions of the equations as before, we arrive at the relationship:

$$Z_m = \frac{b^2\ E_{xd}^2\ K_x^2\ D_x}{S^2}\ Z_x$$

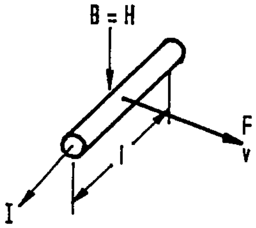
The analog of this transducer becomes:



There is no negative stiffness in this form to counteract the positive stiffness needed to restrain the steady force. This explains why it is not easy to build this type of device with a high low frequency coupling.

A Conducuctor in a Magnetic Air Gap

The conductor in an air gap for the most part applies to the magnetic case and has the condition that the generating mechanism for the signal field is identical with the mechanical terminal. The popular description of this type of device is a 'dynamic' transducer. The usual and apparently most practical form is a coil (current sheet) in an annular magnetic field.



The simple formula connecting electric current to mechanical force is given as:

l is length of the conductor (cm)

$F = .1\ B\ l\ I$ dyne

B is flux density (lines/cm^2)

if the coil is moved in the magnetic field the voltage is:

$E = B\ l\ v\ 10^{-8}$

v is velocity (cm/sec)

putting these in the notation of the previous derivations:

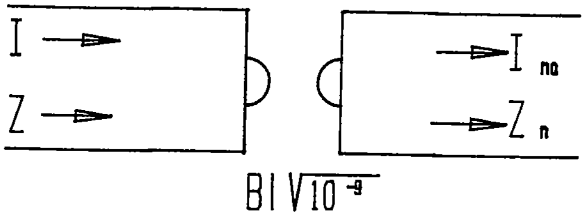
$$E_m = .1\ B\ k_M\ l\ I$$

$$I_m = E/(B\ k_M\ l\ 10^{-8})$$

dividing

$$Z_m = \frac{E_m}{I_m} = \frac{B^2\ k_M^2\ l^2\ 10^{-9}}{(E/I)} = \frac{B^2\ k_M^2\ l^2\ 10^{-9}}{Z_e}$$

At this point a transducer analog can be drawn.



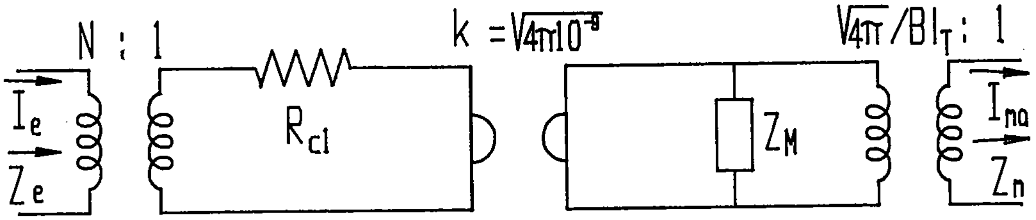
This meets the standards set by most texts and for the purposes of the popular writers seems to be sufficient. More can be done by forcing this into the form of the analogs previously described.

We rewrite N l_T for l $\quad$ l_T is the length of a turn

$$Z = \frac{B^2 \ k_M^2 \ N^2 \ l_T^2 \ 10^{-9}}{Z_m} = 4 \ Pi \ N^2 \ 10\text{-}9 \ . \ \frac{B^2 \ k_M^2 \ l_T^2}{4 \ Pi \ Z_m}$$

Which is recognized as the gyrator and transformer used in the earlier forms. This permits the inclusion of the coil resistance and the inductive effects of the magnetic structure.

> R_{c1} is the resistance of the single turn coil having the same copper volume as the total coil, i.e., a one turn coil.



It can be argued that the shunt capacitances in these analogs should not a part of the transducers and should be considered as part of the associated circuit. It remains in the circuit when the biasing field is reduced to zero.

CALIBRATION

The analogs that have been discussed are linear and passive. In the absence of active devices, the only loss of energy is through resistive dissipation or through the terminals of the circuit. The interfaces serve the purpose of providing the proper interaction between portions of the circuit, where the values attached to the electrical components are numerically convenient in representing the device in question.

The amount of energy in any portion of the circuit is in the same units as was assumed when it was introduced. That is, if the impressed energy is in Joules or Watt-seconds, i.e. volts and amperes, the energy in all other parts of the circuit is in Joules or Watt-seconds. If the input is in cgs units the energy is in ergs in all parts of the circuit. If two sources of energy are used they must be in the same units.

For inputs in electrical units (Joules), responses in cgs portions of the circuit should be increased by 70 dB or multiplied by 3,162 to obtain the value in dynes (or dynes/cm^2).

For inputs in cgs units (Ergs), responses in electrical portions of the circuit should be decreased by 70 db or multiplied by 3.162e-4 to obtain the value in volts or amperes.

APPENDIX A

The gyrator is a concept which, because of its relative infrequent use, is usually only vaguely understood. The following is an attempt to explain the concept.

The ideal gyrator, like the ideal transformer, can be thought of as a passive lossless interface in a circuit where an impedance (Z_2) on one side of the interface appears as different impedance (Z_1) on the other side of the interface. What happens at the interface is controlled by a gyrator factor (or a transformer ratio) k as follows:

Gyrator	**Transformer**
$Z_1 \times Z_2 = k^2$	$Z_1 / Z_2 = k^2$

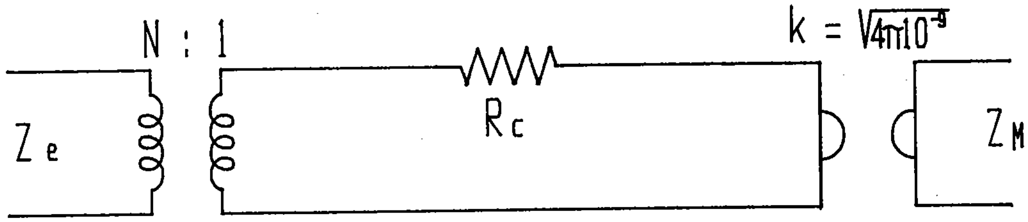
If E_1 and I_1 are observed on the one side of the interface while E_2 and I_2 are observed on the other side of the interface, the above relationships force the conditions listed below.

k^2 / Z_2	=	Z_1	=	$Z_2 \times k^2$
k^2 / Z_1	=	Z_2	=	Z_1 / k^2
$I_2 \times k$	=	E_1	=	$E_2 \times k$
$I_1 \times k$	=	E_2	=	E_1 / k
E_2 / k	=	I_1	=	I_2 / k
E_1 / k	=	I_2	=	$I_1 \times k$

Note that a gyrator is symmetrical; it produces the same relations between voltage and current in either direction through the interface. The transformer has a "sided-ness" to its action. In this definition the factor k for the ideal transformer is the same as saying that on the primary (#1) side there are k times as many turns as on the secondary (#2) side of the transformer.

APPENDIX B

In the analog for the electric-magnetic transducer, the insertion of the transformer, with turns ratio N, which at first may seem like an unnecessary complication, has a useful function in working with a practical analog. In many, if not most, applications the physical volume that can be allotted to the coil which produces the MMF is one of the constraints (although it may be arbitrary.) By considering it as a single turn of the conductor material having the same conductor volume as is allotted to the total coil, it can be introduced into the circuit as a fixed quantity.



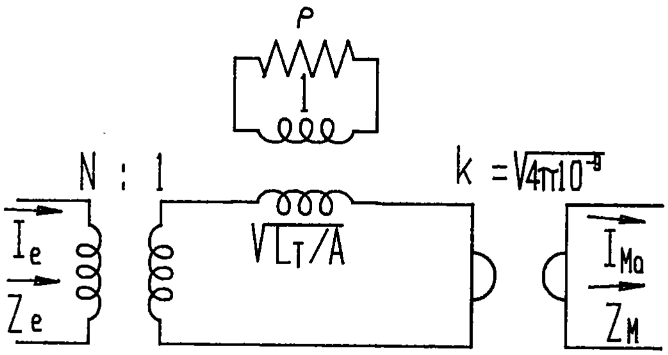
In this form the effects on coil resistance when changing the numbers of turns on the coil are automatically taken into account as well as the effects on sensitivity and impedance.

The turn used for this calculation should be based on the conductor alone. This has a smaller area than the section of the coil because of the wire insulation and a packing factor (for round wires.)

A bit of caution should be exercised when dealing with fine wires as the proportion of the volume devoted to insulation may change if a wide range of "N" is used.

Since wire generally is available in graded sizes where the nominal diameter changes by the 39th root of 92, the area changes a factor of 1.261 : 1. Thus full coils occur at approximately 1.261 : 1 changes in turns and 1.59 : 1 changes in impedance. Usually for other conditions, unless special wire is used, the the wire size and actual volume of copper will have to be reduced to accommodate the required number of turns. This may mean that the value of the coil resistance in practice may be up to 26% higher than predicted.

If in the analog the dimension of the coil are to be a variable, it may be desirable to include a second transformer to introduce new parameters such as:



Where L_t = mean length of turn
A = Area of coil section
= Resistivity of conductor (1.7241×10^{-6} for copper)